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Equity financing constraints and corporate capital structure: a model

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Abstract

Purpose – The "supply-side effect" brought about by the imperfection of the capital market has increasingly been concerned. The purpose of this paper is to study how will the uncertainty of equity financing brought about by the equity financing regulations in emerging capital market affect company's capital structure decisions.

Design/methodology/approach – This paper establishes a theoretical model and tries to introduce equity financing uncertainty into the company's capital structure decision-making. The paper uses mathematical derivation method to get some basic conclusions. Next, in order to characterize the quantitative impact of specific factor on capital structure, numerical solution methods are used.

Findings — The model shows that firm's value would decrease with the uncertainty of equity financing, because of the relationship between firm's future cash and their financing policies. The numerical solution of the model suggests that the uncertainty of equity financing is one of the important factors affecting the choice of optimal capital structure, the greater the uncertainty is, the lower optimal capital structure is.

Originality/value – The research of this paper has certain academic value for further understanding of the issues.

Keywords Capital structure, Corporate financing, Equity financing preference, Equity financing uncertainty

Paper type Research paper

1. Introduction

The modern corporate financing theory originated from the achievement of Modigliani and Miller (1958). Under their strict assumptions, MM reached the conclusion that corporate capital structure was unrelated to corporate value by exploiting the thought of no-arbitrage. Scholars of corporate finance thereafter continuously loosened the assumed preconditions of MM theorem, explored the realistic influencing factors of corporate financing and capital structure and put forward several theoretical hypotheses and a lot of empirical results. However, if we divide the variation of company's capital structure into three levels, namely between-industry variation, within-industry variation and within-firm variation, no matter from which level it is judged, current empirical model has an extremely limited ability to interpret capital structure (Graham and Leary, 2011).



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- (1) assumptions of exogenous cash flow, embracing assumptions of tax, bankruptcy cost, information completeness and complete contract; and
- assumptions of market perfection.

Titman points out that the previous theoretical and empirical studies on corporate financing and capital structure mainly focused on loosening the assumptions of cash flow exogenesis in MM hypotheses and ignored the assumptions of market perfection. But in reality, financial market is imperfect and has various frictions or constraints. This leads to the estrangement between academic circles and practitioners in the cognition of financing decision-making and capital structure. The former spends plenty of energy on cash flow assumptions, whereas the latter pays more attention to the imperfection of market. For this reason, Titman appeals to the studies on corporate financing and capital structure for more focus on the imperfection of capital market and calls this imperfection brought about by the features of capital suppliers the "supply-side effect". With respect to the future direction of studies on capital structure. Graham and Leary (2011) points out again that the attention paid to the "supply-side effect" is too little and appeals for enhancement in this regard in future studies.

The first theoretical hypothesis focusing on the "supply-side effect" was "market timing hypothesis". In the wake of the asset pricing field's doubt about the "efficiency market hypothesis", researchers started to pay attention to the impact of inefficient market on corporate investment/financing decision-making and capital structure. Stein (1996) studies the investment/financing behaviors of company in the case of inefficient market and rational enterprise managers. His model indicates that in an inefficient market, the manager of the company can exploit the inefficiency of market to reasonably arrange financing to obtain benefit. Baker and Wurgler (2002) formally put forward the market timing hypothesis for the first time; along with the price changes in stock market, there is the best financing timing or financing opportunity window for the company, and most companies should make additional issuance in the overall rise stage of stock market or the period when their own stock price is rising high. The market timing hypothesis of enterprise financing is empirically supported in the Western capital market. In recent years, some domestic studies have also focused on the effect of market timing factor on enterprise's financing behavior and capital structure and find that market timing does play a significant role in equity financing of company (Liu et al., 2005, 2006; Liu and Li, 2005; Wang et al., 2005).

Since the market timing is one kind of manifestation of the "supply-side effect" of stock market, the debt market also shows the "supply-side effect". Murfin (2012) points out that banks write tighter contracts than their peers after suffering payment defaults to their own loan portfolios, even when defaulting borrowers are in different industries and geographic regions from the current borrower, it will also be implicated by such a supply-side effect; borrowers who are most dependent on the relationship aspect of the bank market are also most prone to receive stricter contracts from affected lenders.

The study on the "supply-side effect" is in the stage of preliminary development, and a uniform framework has yet to form. Besides, these expansions are based on the situation of developed capital market. Myers (2003) had pointed out that a majority of capital structure theories were constructed based on the US listed companies, but due to different conditions of capital market, all theories had their own applicable assumptions, so established theories and interpretations did not necessarily apply to emerging capital market, for which, as a matter of fact, the condition of capital supply lags far behind that of the Western mature capital market on various aspects, such as the variety of financing instruments, government regulation environment, etc. The "supply-side effect" is more prominent in the emerging capital market.

In the development process of China's stock market over the past more than 20 years, strict regulations on initial public offering and refinancing are still followed today, giving rise to the relatively high uncertainty of corporate equity financing. Equity financing regulation is reflected in two aspects. First, stock issuance regulation: under the standard of "high unity of development, normalization and market bearing capacity" in the stock market of China, the government implements comparatively rigorous regulation on securities issuance: a company needs to satisfy financial thresholds (net return on equity, cash dividend distribution, etc.) first of all for the purpose of issuing securities; next, a listed company complying with issuance access conditions is also subject to administrative regulations on issuance pricing, issuance timing, issuance tempo, issuance scale, and so on. For example, the regulatory authority will suspend stock issuance in the period of stock market downturn or due to a special need and loosen the regulation on issuance tempo when the stock market goes up.

Second, the government, based on the needs for macro-control, industrial development and stock market stability, limits and even suspends normal supply of financial products. Even though an enterprise meets the conditions for stock issuance, it is not for sure that it can obtain equity capital when needing equity financing. For instance, the China Securities Regulatory Commission (CSRC) will limit the listing and refinancing of real estate enterprises to coincide with the national regulatory policies of real estate. During full-circulation share reform, the CSRC had once shut the door to stock issuance. Since November 2012, IPO had been discontinued again.

From a macroscopic view, the regulation and limitation on stock issuance are helpful for facilitating the sustainable steady development of emerging securities market (Zhu and Cheng, 2005) and improving the effectiveness of macroeconomic regulation and control. But for enterprises, under the situation of increasingly fierce product market competition, regulation and limitation bring about great uncertainty for corporate equity financing, and it is difficult for a company to determine whether it can smoothly raise capital through stock market in the future. This uncertainty of future equity financing is obviously an important content of the "supply-side effect". (There is generally no such an effect in the developed capital market. For example, there is "rapid refinancing system" in many developed capital markets.)

How will the uncertainty brought about by the regulation and limitation on financing in stock market affect the equity financing behavior and capital structure of company? Wang *et al.* (2011) find from the study with the data of China's listed companies that the changes of refinancing regulatory policies for listed companies significantly affect the optimal capital structure of listed companies, but on account of the difficulty of variables design, their study does not point out the specific directions

constraints

of the affect of refinancing regulation on capital structure. For instance, by loosening or Equity financing tightening the refinancing policy, will the optimal capital structure of company become higher or lower? Furthermore, what is the influence mechanism behind it? Studies on this aspect have yet to be seen so far[1].

The innovation of the paper is that by establishing a mathematical model, it depicts the "supply-side effect" of equity financing (reflected as the probability of equity financing in the future) on company brought about by equity financing regulation and its affect on capital structure decision-making in the operating process of company. It is found through the model that the uncertainty of external equity financing of company will result in value loss of company's shareholders, which increases along with the magnification of uncertainty; additionally, the uncertainty of financing in stock market will also affect the choice of optimal capital structure of company, and the greater the uncertainty is, the lower optimal capital structure is. The model of the paper is helpful for understanding the influence of external equity financing environment on company's capital structure.

The structure of the paper is arranged as follows: in Section 2, we establish a mathematical model of financing regulation in stock market and corporate optimal investment/financing decision-making; in Section 3, the influence of several main variables of the model on the choice of optimal capital structure of company is depicted with the method of numerical analysis; Section 4 is the conclusion of the paper.

2. Model

2.1 Basic assumptions

We assume the operation objective of company is the value maximization of all shareholders, and the cash flow of company is related to investment. Most models studying capital structure assume the future cash flow of company is exogenous and unrelated to financing decision-making (Modigliani and Miller, 1958; Hackbarth et al., 2006; Strebulaev, 2007), implicitly assume the company can raise funds in capital market for investment opportunity with NPV > 0 at any time with no friction. However, in fact, when capital market is imperfect and there are constraints on corporate financing, the available funds of company will inevitably affect the investment ability of company and thereby the cash flow of company.

Make the investment of company in stage $t I_b$; this investment produces profits in stage t+1; suppose the payoff on investment is $a_{t+1}f(I_t)$, among which f is increasing function and satisfies the principle of diminishing marginal returns. That is to say, f(0) = 0, f' > 0, and f'' < 0; a_{t+1} is a random variable, which means the uncertainty factor affecting the return on investment at the level of macro-economy or company. In stage t, a_t is given information.

Suppose the liabilities of company in stage t are D_b and the interest rate of liabilities is r_D ; τ represents the tax rate of company; then the payoff on equity investment of company in stage t+1 predicted in stage t, π_{t+1} , can be shown as:

$$\pi_{t+1} = (a_{t+1}f(I_t) - I_t - D_t r_D)(1 - \tau) \tag{1}$$

Define $\delta_t = -e_t$; δ_t refers to the dividend given by the company to shareholders in stage t (when $\delta_t < 0$, it means that the company has made external equity financing, which can be understood as negative dividend). According to the dividend discount model, the objective of company at present moment (t = 1) can be shown as:

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$$\max V = \mathbb{E}_1 \left[\sum_{t=1}^T \frac{\delta_t}{(1 + r_E)^{t-1}} \right] \tag{2}$$

 \mathbb{E} refers to the expectation operator for dividend, r_E represents the cost of equity use of company, and $r_E > r_D$. Here, we do not consider the effect of behaviors like management confidence on the financial issues of company.

2.2 Financing constraints in financial market

The constraint of bank loan on company is considered in the first place. The repayment of capital with interest of bank loan is a hard constraint on company, but in the event of bankruptcy and liquidation, the company only bears limited responsibilities for the bank loan. Therefore, the bank always makes certain limitation on the debt ratio of company. With reference to Baker *et al.* (2003), we assume the company will face the limit of the highest debt ratio (\bar{L}) set by the bank while using bank loan, namely:

$$\frac{D_t}{D_t + E_t} \le \bar{L} \tag{3}$$

Next, the financing constraint in stock market confronting the company is considered. This makes it uncertain for the company to obtain equity financing in stage t. Let whether the company can obtain equity financing hinge on an independent and identically distributed random state variable b_t , which complies with (0-1) distribution; the value of b_t is 1 with a probability p (indicating the company can obtain equity financing in stock market) and 0 with probability 1 - p (indicating the company cannot obtain equity financing in stock market, i.e. $\delta_t \ge 0$); then the constraint can be shown as:

$$\delta_t \ge 0, \quad \text{if } b_t = 0 \tag{4}$$

The financing constraint most immediately affects the company's available capital and investment. In the model herein, the available capital of company can be shown as $E_t + D_t$. An intuitive constraint on investment is that the investment of company in current period cannot exceed current available capital, namely:

$$I_t \le E_t + D_t \tag{5}$$

Formulas (3) through (5) are the financing constraints on company during its normal operation (i.e. in case of $E_t > 0$). When the net assets of company are negative (i.e. in case of $E_t \leq 0$), the impact of financing constraints on company will be greater. Under such a circumstance, the company actually sinks into financial distress and is on the verge of bankruptcy. Thus, it is hard to raise capital and make investment, let alone dividend policy. Under such a condition, the constraints on company can be shown as:

$$\delta_t = 0, \ D_t = 0, \ I_t = 0, \ if E_t < 0$$
 (6)

Under the objective and the constraints mentioned above, we are unable to solve the model by directly using Lagrange multiplier method, because due to the existence of random variables, all the variables may not be necessarily derivable everywhere.

To solve the model, we need to analyze it on the basis of the distribution of a_t .

A static model is first considered: at the time when t = 1, the company makes an investment decision I_1 and financing decisions D_1 and E_1 . At the time t = 2, the company is liquidated. The simple schematic diagram is shown in Figure 1.

At the time t=2, if the net assets of company is negative (i.e. $a_2f(I_1)-I_1-D_1r_D+E_1<0$), the shareholder payoff will be 0 (the limited liability nature of company limited by shares). Besides, if the profit of company is negative (i.e. $a_2f(I_1)-I_1-D_1r_D<0$), the government cannot levy the tax. Then, the shareholder objective of company can be shown as:

$$\max V = -E_1 + \int_{a_2^*}^{a_2^*} \frac{E_1 + a_2 f(I_1) - I_1 - D_1 r_D}{1 + r_E} dG(a_2)$$

$$+ \int_{a_2^*}^{\infty} \frac{E_1 + [a_2 f(I_1) - I_1 - D_1 r_D](1 - \tau)}{1 + r_E} dG(a_2)$$
Subject to: $I_1 \le E_1 + D_1 \quad \frac{D_1}{D_1 + E_1} \le \bar{L}$

where G() represents the distribution function:

$$a_2^* = \frac{I_1 + D_1 r_D}{f(I_1)}$$

and:

$$a_2^{**} = \frac{I_1 + D_1 r_D - E_1}{f(I_1)}.$$

By rearranging and solving the objective above, we can reach the conclusion that the optimal investment I_1 of company satisfies:

$$f'(I_1) = \frac{(1+r_E)(1-\bar{L}) + \bar{L}(1+r_D)\left(1-G\left(a_2^{**}\right)\right) - \tau(1+\bar{L}r_D)\left(1-G\left(a_2^{*}\right)\right)}{\int_{a_2^{**}}^{\infty} a_2 dG(a_2) - \tau \int_{a_2^{*}}^{\infty} a_2 dG(a_2)}$$
(8)

And the following proposition can be obtained:

Proposition 1. In a static model, due to the absence of the going-concern pressure of company, the investment amount of company satisfies formula (8); the financing amount of company satisfies $E_1 + D_1 = I_1$, namely that the financing amount exactly satisfies its investment demand. Meanwhile, the debt ratio of company will reach the upper limit \bar{L} .



Figure 1. Static model of company's investment and financing decisions

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In other words, because at the time t=2 the company faces liquidation, it will avoid using equity as far as possible and exploit liabilities to the upper limit to the greatest extent during financing at the time t=1 (this is consistent with the capital structure theory of MM (1963) when only tax is taken into consideration). Under such a condition, equity financing constraint does not work in the static model.

2.4 A three-stage dynamic model

On the basis of the static model, we can consider a three-stage dynamic model to study how the company comprehensively selects its optimal capital structure in combination with its current investment demand and future development in a dynamic process.

The same with the two-stage static model, at the time when t=1, the company makes an investment decision I_1 and financing decisions D_1 and E_1 ; at the time t=2, state variable a_2 of the return on investment is realized, and state variable b_2 of whether the company can implement external financing is also realized. On this ground, the company needs to make investment decision I_2 and financing decisions D_2 and E_2 in stage 2; at the time t=3, the company is liquidated. The simple schematic diagram is shown in Figure 2.

To obtain the optimal investment and financing demands of company at the time t=1, we can employ the backward induction, that is, first determining the optimal investment and financing demands of company at the time t=2, and then working out the optimal investment and financing demands at the time t=1.

the optimal investment and financing demands at the time t=1. Use I_2^* , D_2^* and E_2^* to, respectively, represent the optimal investment and financing decisions of company at the time t=2. It can be known through static analysis that when t=2, the investment and financing decisions of company satisfy $D_2^* + E_2^* = I_2^*$, and company's capital structure will reach the upper limit \bar{L} , i.e. $D_2^* = I_2^* \bar{L}$. Meanwhile, I_2^* is the function of random variable a_3 and satisfy:

$$f'\Big(I_2^*\Big) = \frac{(1+r_E)(1-\bar{L}) + \bar{L}(1+r_D)\Big(1-F\Big(a_3^{**}\Big)\Big) - \tau(1+\bar{L}r_D)\Big(1-F\Big(a_3^*\Big)\Big)}{\int_{a_3^*}^\infty a_3 dF(a_3) - \tau \int_{a_3^*}^\infty a_3 dF(a_3)}$$

where:

$$a_3^* = \frac{I_2 + D_2 r_D}{f(I_2)}$$

and:

$$a_3^{**} = \frac{I_2 + D_2 r_D - E_2}{f(I_2)}.$$

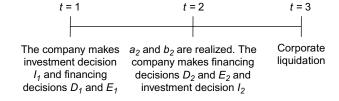


Figure 2. A dynamic model of company's investment and financing decisions

Based on the optimal investment and financing demands of company at t = 2, we can Equity financing analyze the investment behavior and shareholder income of company at this moment. Here, we need to discuss them under the following cases.

Case 1. If $E_1 + [a_2 f(I_1) - I_1 - D_1 r_D](1 - \tau) \ge E_2^*$, it means the shareholder payoff generated by the investment of company in stage 1 can meet the optimal equity demand of company at the time t=2. In such a case, whether equity financing can be realized in capital market has no affect on corporate investment. As a result, the funds of company at the time t=2 can satisfy the optimal investment amount I_2^* , so the investment amount of company at t=2 is I_2^* , and the equity is E_2^* . It can be known from the conclusion in Section 2.3 that because the company is liquidated at the time t = 3, the company will reserve no more equity after its investment demand is satisfied; thus the dividend of company at t = 2 is $E_1 + [a_2 f(I_1) - I_1 - D_1 r_D](1 - \tau) - E_2^*$. At the same time, the present value to t = 2 of the equity payoff at t = 3 is $V_2(I_2^*, E_2^*, D_2^*) + E_2^*$, where function V_2) represents the value function of shareholder income generated by corporate investment, following the result of formula (8).

Case 2. If $E_1 + [a_2 f(I_1) - I_1 - D_1 r_D](1 - \tau) < 0$, it means the company suffers a serious loss and becomes insolvent at the time t = 2. In such a case, no matter whether future investment can bring positive net present value, current negative net assets are a kind of "burden". At the moment, the optimal decision of company is bankruptcy and reorganization[2]. The incomes of shareholders brought by the company when t=2and t = 3 are both zero.

Case 3. If $0 \le E_1 + [a_2 f(I_1) - I_1 - D_1 r_D](1 - \tau) < E_2^*$, it means although the company is not insolvent at the time t = 2, its equity cannot meet its optimal equity demand. In such a case, whether equity financing can be realized in market appears to be crucial. If b = 1, namely that the company can realize equity financing in capital market, meaning the company does not have the problem of insufficient equity financing, the available funds of company can still satisfy the optimal investment amount I_2^* at the time t=2; therefore the investment amount of company is I_2^* , and the equity is E_2^* at t=2. It can also be known from the aforesaid conclusion that because the company is liquidated at the time t = 3, the company will reserve no more equity after its investment demand is satisfied; thus the dividend of company at t = 2 is $E_1 + [a_2 f(I_1) - I_1 - D_1 r_D](1 - \tau) - E_2^*$, and at the same time, the present value to t=2 of the equity payoff at t=3 is $V_2(I_2^*,E_2^*,D_2^*)+E_2^*$. But if b=0, namely that the company cannot realize equity financing in capital market, the company will be confronted with the problem of insufficient equity. It can be known from the optimality of I_2^* , D_2^* and E_2^* that the equity investment of company can only be in a "suboptimum" state, represented by superscript "**"; then, the equity investment amount of company $E_2 \leq E_1 + [a_2f(I_1) - I_1 - D_1r_D](1-\tau)$, the dividend of company at t=2 is $E_1 + [a_2f(I_1) - I_1 - D_1r_D](1-\tau) - E_2$, and meanwhile, the present value to t=2 of the equity payoff at t=3 is $V_2\left(I_2^{**},D_2^{**},E_2^{**}\right)+E_2^{**}$, where I_2^{**} and D_2^{**} represent suboptimum total investment level and debt level, respectively.

By combining the three cases above and noticing the probability that the company cannot raise money in capital market is 1-p, we use V_2^* and V_2^{**} to represent

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 $V_2(I_2^*, E_2^*, D_2^*)$ and $V_2(I_2^{**}, D_2^{**}, E_2^{**})$, respectively; then, the shareholder objective function of company at the time t = 1 can be shown as:

$$\max V_{1} = \frac{E_{1} - (I_{1} + D_{1}r_{D})(1 - \tau)}{1 + r_{E}} \left(1 - G\left(a_{2}^{*}\right) \right)$$

$$+ \int_{a_{2}^{*}}^{\infty} \frac{V_{2}^{*} + (1 - \tau)a_{2}f(I_{1})}{1 + r_{E}} dG(a_{2})$$

$$- (1 - p) \int_{a_{2}^{*}}^{a_{2}^{*}} \frac{V_{2}^{*} - V_{2}^{*}}{1 + r_{E}} dG(a_{2}) - E_{1}$$
Subject to: $I_{1} \leq E_{1} + D_{1}$; $\frac{D_{1}}{D_{1} + E_{1}} \leq \bar{L}$;
$$E_{2}^{**} \leq E_{1} + [a_{2}f(I_{1}) - I_{1} - D_{1}r_{D}](1 - \tau) \quad a_{2}^{*} = \frac{I_{1} + D_{1}r_{D} - E_{1}/(1 - \tau)}{f(I_{1})}$$
;
$$a_{2}^{**} = \frac{I_{1} + D_{1}r_{D} + \left(E_{2}^{*} - E_{1}\right)/(1 - \tau)}{f(I_{1})}$$

It is worth noting that under the constraint condition $E_2^{**} \leq E_1 + [a_2f(I_1) - I_1 - D_1r_D](1-\tau)$, any a_2 has a E_2^{**} corresponding to it, so E_2^{**} is the function of a_2I_1 , D_1 and E_1 that make the value of formula (9) maximum are the optimal investment and financing decisions at the time t=1. In formula (9), V_2^{**} shows the shareholder value of company is in the suboptimum state, whereas V_2^* shows this value is in the optimal state. It can be known from the analysis of case 3 that $V_2^{**} < V_2^*$; in addition, because $a_2^* < a_2^{**}$:

$$\int_{a_0^*}^{a_2^{**}} \frac{V_2^* - V_2^{**}}{1 + r_E} dG(a_2) \ge 0.$$

Thus, we obtain the following proposition:

Proposition 2. In the dynamic model, the uncertainty of external equity financing in the next stage will bring about value loss of shareholders. The lower the probability p that the company can obtain external equity financing, the larger the value loss of shareholders is brought about by financing uncertainty.

To work out the optimal I_1 , D_1 and E_1 in formula (9), we can write the Lagrangian equation of formula (9) and constraint condition and solve the first-order optimal condition. Since it is difficult to work out the analytical solution of formula (9), we will resort to the method of numerical solution in the next section.

3. Numerical analysis

In the dynamic model, although we give the determining equation of optimal investment and financing decisions, as well as the solving method, it is very hard to obtain the explicit solution of model in view of its complexity. In order to observe the result of model Equity financing in a more intuitive way, we can give the specific function form of f(I) and the specific distribution form of a, and then the optimal L and I_1 with the method of numerical

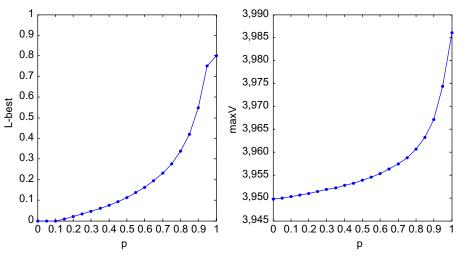
We might make $f(I) = 20\sqrt{I}$; then f(I) satisfies assumptions f(0) = 0, f' > 0, and f'' < 0[3].

The poorest condition of corporate investment is af(I) = 0, so we might assume that the random distribution a of the return on investment in the future is geometrical normal distribution. In addition, because a_1 is a known variable at the time when t=1, for simplicity, we might suppose $a_1 = 1$. $a_{t+1} = a_t * \tilde{\eta}_{t+1}$, where $\ln \tilde{\eta}_{t+1}$ complies with the normal distribution of $N(\mu_{t+1}, \sigma_{t+1}^2)$, in which μ_{t+1} represents the expected growth rate of the return on investment from stage t to stage t+1 and σ_{t+1}^2 measures the risk in future economy.

3.1 Equity financing probability and company's capital structure

We first of all pay attention to the effect of probability p that the company can obtain external equity financing at the time when t=2 on the optimal capital structure. Make parameters $\bar{L}=80\%$, $\tau=25\%$, $r_E=10\%$, $r_D=5\%$, $\mu_2=\mu_3=1$, and $\sigma_2=\sigma_3=0.3$ [4], substitute them into the original equation, and solve the equation; then we obtain the changes of optimal capital structure along with p, shown as in Figure 3.

In the left subgraph of Figure 3, we can find that with the given parameters, when b approaches to zero, the optimal capital structure of company is also zero, and as p increases, the numerical value of optimal capital structure increases accordingly. When p=1, the optimal capital structure of company is at the point $\bar{L}=80\%$.



Notes: Parameters: $f(I) = 20\sqrt{I}$, $r_E = 10\%$, $r_D = 5\%$, $\bar{L} = 80\%$, t = 25%, $\mu_2 = \mu_3 = 1$, $\sigma_2 = \sigma_3 = 0.3$

Figure 3. Equity financing probability vs optimal capital structure and shareholder value The result of this numerical solution indicates that the larger the financing constraint in stock market (in the model, reflected by the smaller probability p that the company can obtain external equity financing at t = 2) is, the lower the optimal capital structure based on enterprise value maximization is.

The right subgraph of Figure 3 intuitively shows us the effect of probability p that the company can obtain external equity financing at t = 2 on the current shareholder value of company. It can be observed from the subgraph that along with the increase of p, shareholder value increases as well. To be more intuitive, the smaller the uncertainty of corporate financing in stock market in the next stage is, the larger the shareholder value is. This result is consistent with the conclusion of Proposition 2.

3.2 Debt ratio limit, equity financing probability and optimal capital structure Based on the results above, we can also investigate the impact of bank's limit to the highest debt ratio of company on the optimal capital structure of company and then the effect of the combined change of highest debt ratio limit and p on corporate capital structure, which helps us know the marginal effect of p on capital structure in different cases.

To make it comparable with the results of Figure 3, we still set parameters as follows: $\tau = 25\%$, $r_E = 10\%$, $r_D = 5\%$, $\mu_2 = \mu_3 = 1$ and $\sigma_2 = \sigma_3 = 0.3$. Table I and Figure 4 give the optimal debt ratios of company in case of different values of p in the range of \bar{L} from 65 percent to 90 percent.

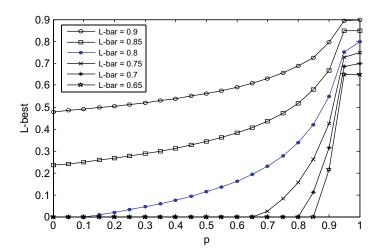
In Table I, with other parameters given, we provide the optimal shareholder values of company corresponding to assigned p and \bar{L} . It can be seen in the table that the shareholder value of company tends to rise in the direction of "south-east". That is to say, the shareholder value of company rises gradually with the increase of p. On the other side, along with the increase of \bar{L} , the shareholder value of company also rises, indicating that \bar{L} is also one of the factors influencing the shareholder value of company.

The results in Table I are shown in Figure 4 in a more comprehensive and intuitive manner. It can be observed from the figure that as \bar{L} decreases (namely that the bank

	L-bar						
P	65 (%)	70 (%)	75 (%)	80 (%)	85 (%)	90 (%)	
0.0	0.00	0.00	0.00	0.00	23.46	47.89	
0.1	0.00	0.00	0.00	0.00	24.95	49.12	
0.2	0.00	0.00	0.00	2.09	26.75	50.45	
0.3	0.00	0.00	0.00	4.58	28.78	52.06	
0.4	0.00	0.00	0.00	7.60	31.24	53.91	
0.5	0.00	0.00	0.00	11.37	34.30	56.20	
0.6	0.00	0.00	0.00	16.26	38.19	59.12	
0.7	0.00	0.00	2.55	23.07	43.55	63.04	
0.8	0.00	0.00	15.74	33.69	51.79	68.92	
0.9	21.43	31.38	42.59	54.84	66.75	79.72	
1.0	65.00	70.00	75.00	80.00	85.00	90.00	

Table I. Impact of debt ratio limit and equity financing probability on optimal capital structure

Notes: Parameters are set as follows: $f(I) = 20\sqrt{I}$, $r_E = 10\%$, $r_D = 5\%$, $\tau = 25\%$, $\mu_2 = \mu_3 = 1$ and $\sigma_2 = \sigma_3 = 0.3$; L-bar represents the limit of highest debt ratio, and p represents the probability of equity financing in market in the next stage; results in the table are the optimal debt ratios corresponding to L-bar and p



Equity financing constraints

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Figure 4.
Effect of debt ratio limit
and equity financing
probability on optimal
capital structure
of company

tightens the limit to the debt ratio of company), the line representing the optimal shareholder value of company gradually declines, showing the financing friction brought about by debt ratio limit from the bank will also reduce the shareholder value of company. Besides, with every given \bar{L} , along with the increase of p, the shareholder value of company gradually rises, which is coincident again with the conclusion of Proposition 2.

Additionally, through observing the shapes of all curves, we can find that when \bar{L} is relatively high, the curve is relatively gentle, whereas when \bar{L} is lower, the curve is steeper. This result indicates that along with the increase of \bar{L} , the sensitivity of the optimal shareholder value of company to p decreases; similarly, along with the increase of p, the sensitivity of the optimal shareholder value of company to \bar{L} also decreases.

3.3 Growth opportunity, equity financing probability and optimal capital structure. In the model herein, random variable \tilde{a}_2 is an important variable. Its characteristics may be an important factor influencing the optimal capital structure. In our assumption about \tilde{a} , μ_{t+1} , as an important characteristic variable of \tilde{a} , represents the expected growth rate of the return on investment from stage t to stage t+1, so it is necessary to investigate the combined change of μ_{t+1} and p (representing future growth) on the optimal decision-making of company.

Similarly, to make it comparable with the results above, we still set parameters as follows: $\tau = 25\%$, $r_E = 10\%$, $r_D = 5\%$, $\bar{L} = 80\%$ and $\sigma_2 = \sigma_3 = 0.3$. Table II gives the optimal capital structures of company in case of different values of p in the range of μ gradually increasing from 0.5 to 1.2.

It can be seen in Table II that in each column, as p increases, the optimal capital structure rises, which coincides with the conclusion of Section 3.1 again; in each row, as μ increases, the optimal capital structure shows a trend of decline – this result is consistent with the theoretical study of Myers (1977). According to Myers, when the future growth opportunity is higher, the company has more real options; if the company adopts debt financing in such case, it means that the company may give up these options, because such investment transfers wealth from shareholders to the creditor.

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Table II. Effect of growth opportunity and equity financing probability on optimal capital structure

	μ							
Þ	0.5 (%)	0.6 (%)	0.7 (%)	0.8 (%)	0.9 (%)	1.0 (%)	1.1 (%)	1.2 (%)
0.0	70.54	62.45	51.69	37.82	20.20	0.00	0.00	0.00
0.1	71.70	63.73	53.15	39.38	21.89	0.00	0.00	0.00
0.2	73.05	65.15	54.71	41.15	23.87	2.09	0.00	0.00
0.3	74.00	66.79	56.63	43.23	26.13	4.58	0.00	0.00
0.4	76.28	68.78	58.81	45.73	28.89	7.60	0.00	0.00
0.5	77.15	70.99	61.52	48.76	32.29	11.37	0.00	0.00
0.6	80.00	73.87	64.96	52.67	36.66	16.26	0.00	0.00
0.7	80.00	77.03	69.47	57.99	42.69	23.07	0.00	0.00
0.8	80.00	80.00	75.36	66.00	51.98	33.69	10.38	0.00
0.9	80.00	80.00	80.00	76.88	69.31	54.84	35.24	10.15
1.0	80.00	80.00	80.00	80.00	80.00	80.00	80.00	80.00

Notes: Parameters are set as follows: $f(I) = 20\sqrt{I}$, $r_E = 10\%$, $r_D = 5\%$, $\tau = 25\%$, $\bar{L} = 80\%$ and $\sigma_2 = \sigma_3 = 0.3$; μ represents the growth of future corporate income, and p represents the probability of equity financing in market in the next stage; results in the table are the optimal debt ratios corresponding to μ and p

The results in Table II are shown in Figure 5 in a more detailed and intuitive way. The graphical results clearly show us that consistent with the result above, the financing friction in stock market is still an important factor influencing the optimal capital structure of company: with other parameters given, the larger the financing friction in stock market is (reflected by smaller p), the lower the optimal capital structure of company is. Meanwhile, it can be seen from the positions of all curves that the future corporate growth also significantly influences the optimal capital structure of company — along with μ increases, the curve of optimal capital structure declines gradually. This result preliminarily demonstrates that in our three-stage dynamic model, when the company makes a decision on current optimal capital structure, the future corporate growth is one of the important factors that should be considered; specifically, the higher the future corporate growth is, the lower the current optimal capital structure of company is. It is especially worth noting that

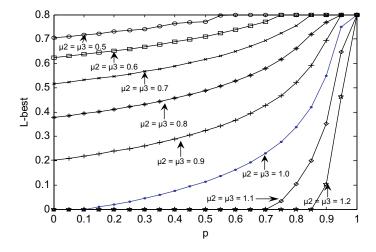


Figure 5.
Effect of growth opportunity and equity financing probability on optimal capital structure

when μ is relatively large and p relatively low, the optimal capital structure of company is Equity financing in the state of zero capital structure; on the contrary, when μ is relatively small and p relatively high, the optimal capital structure of company is at the highest limit position (in this result, zero capital structure is helpful for interpreting the "financial conservative" behavior while the highest capital structure for "financial radical" behavior).

3.4 Income volatility, equity financing probability and optimal capital structure of It is observed that in our assumption about \tilde{a} , σ_{t+1}^2 represents the fluctuation degree of the return on investment from stage t to stage t+1, therefore it is another important characteristic variable of \tilde{a} . As a result, it is necessary to investigate the combined change of σ_{t+1} and p (representing future income volatility) on the optimal decision-making of company.

Similarly, to make it comparable with the results above, we still set parameters as follows: $\tau = 25\%$, $r_E = 10\%$, $r_D = 5\%$, $\bar{L} = 80\%$ and $\mu_2 = \mu_3 = 1$. Table III describes the optimal capital structures of company in case of different values of p in the range of σ increasing from 0.1 to 0.5.

It can be seen from Table III that the optimal capital structure tends to rise along the direction of "left bottom". To be specific, in every column, as p increases, the optimal capital structure gradually rises, which coincides with the aforesaid conclusion again; in every row, as σ increases, the optimal capital structure shows a trend of decline.

The results in Table III are shown in Figure 6 in a more detailed and intuitive way. The graphical results clearly show that consistent with the result above, the financing friction in stock market is still an important factor influencing the optimal capital structure of company: with other parameters given, the larger the financing friction in stock market is (reflected by smaller p), the lower the optimal capital structure of company is. Meanwhile, it can be seen from the positions of all curves that the future income volatility of company also significantly influences the optimal capital structure of company – along with σ increases, the curve of optimal capital structure declines gradually. This result preliminarily demonstrates that in our three-stage dynamic

			σ			
Þ	0.1 (%)	0.2 (%)	0.3 (%)	0.4 (%)	0.5 (%)	
0.0	32.93	15.60	0.00	0.00	0.00	
0.1	34.03	17.20	0.00	0.00	0.00	
0.2	35.29	19.00	2.09	0.00	0.00	
0.3	36.78	21.13	4.58	0.00	0.00	
0.4	38.60	23.71	7.60	0.00	0.00	
0.5	40.88	26.91	11.37	0.00	0.00	
0.6	43.93	31.10	16.26	1.54	0.00	
0.7	48.33	36.95	23.07	8.97	0.00	
0.8	55.64	46.25	33.69	20.43	6.74	
0.9	72.42	65.55	54.84	42.77	30.31	
1.0	80.00	80.00	80.00	80.00	80.00	

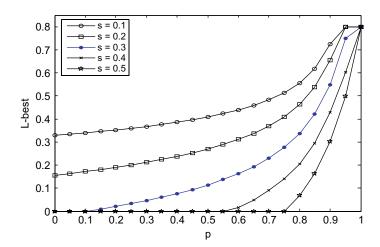
Notes: Parameters are set as follows: $f(I) = 20\sqrt{I}$, $r_E = 10\%$, $r_D = 5\%$, $\tau = 25\%$, $\bar{L} = 80\%$ and $\mu_2 = \mu_3 = 1$; σ represents the future income volatility of company, and p represents the probability of equity financing in market in the next stage; results in the table are the optimal debt ratios corresponding to σ and p

Table III. Effect of income volatility and equity financing probability on optimal capital structure

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Figure 6. Effect of income volatility and equity financing probability on optimal capital structure



model, when the company makes a decision on current optimal capital structure, apart from several important factors mentioned above, the future income volatility is one of the important factors that should be considered; specifically, the higher the future income volatility is, the lower the current optimal capital structure of company is. It is worth particularly noting that when σ is relatively high and p relatively low, the optimal capital structure of company is in the state of zero capital structure.

4. Conclusion

Through a model and numerical analysis, the paper discusses the effect of the equity financing probability on the optimal capital structure of company. The study conclusion demonstrates:

- In the static model, due to the absence of the going-concern pressure of company, the optimal financing always uses liabilities as far as possible to maintain the optimal capital structure at the upper limit. This is consistent with the conclusion of MM (1963) when only tax is taken into consideration.
- In the dynamic model, the company needs to consider the operation in the next stage. Since the future cash flow of company is related to its current investment and financing activities, the uncertainty of company's external equity financing will impact the shareholder value. Specifically, the smaller the probability that the company can realize external equity financing in the next stage, the greater the loss of shareholder value will be caused.
- The numerical solution of dynamic model shows that with model parameters set, the probability of financing in stock market has a significant effect on the optimal capital structure of company: the smaller the probability that the company can realize financing in stock market, the lower its optimal capital structure is; next, the loan amount limit of company from the bank is another factor influencing the optimal capital structure. The stricter the bank limit of loan amount is, the lower the current optimal capital structure of company is.

The study conclusion of this paper can add a new factor for the interpretation of equity financing financing preference of A-share listed companies. Such companies mostly belong to competitive industries in the stage of rapid growth and expansion and need to make continual investment on the premise of steady finance. However, stock issuance regulation increases the uncertainty of corporate financing in stock market. Therefore, as to entity companies, the opportunity of equity financing is very precious: by seizing this opportunity, the company is able to reserve equity funds to cope with the possible equity uncertainty in future, strengthen the ability to compete and the flexibility to grasp development opportunities, including investment in necessary projects or merger and acquisition, and maintain a high credit rating; especially under the circumstances of uncertain business environment, fierce competition and strict financing conditions in stock market, the company can gradually reduce financial leverage and capital cost and increase shareholder value by means of subsequent debt financing. For these reasons, listed companies prefer to equity financing and strive to meet the access conditions of stock issuance and obtain the qualification of stock issuance even through earnings management. Such a behavior cannot be totally attributed to low financing cost, corporate governance, insider control and other corporate factors, but also involves the factor of rational reaction to the equity financing uncertainty brought about by stock market conditions and financing regulations.

The study in this paper also has some enlightenment significance for regulatory authorities. One of the aims of regulatory authorities of capital market by taking strict regulatory measures for equity refinancing of listed companies is to allocate limited capital to better investment projects by means of government intervention, thereby improving the allocation efficiency of capital. But seen from actual operation, such a regulatory mode is liable to make company "intentionally hoard" equity capital and thereby reduce the utilization efficiency of equity capital. As a result, regulatory authorities need to further think about how to impel listed companies to improve the use efficiency of equity capital with more effective measures.

Notes

- 1. In the academic circle of corporate finance, there are many studies focusing on the effect of financing constraints on capital budget, dividend policy, cash reserve and risk management. For example, it is shown by the result of the general model established by Almeida et al. (2011) that the greater the future financing constraints are, the more the company currently tends to invest projects with shorter payback period and lower risk, as well as assets with good investment liquidity and high mortgage value. But the financing constraints mentioned by these studies are generally defined as the condition that current cash flow cannot meet current investment demand, but still implicitly assume the company is able to acquire financing from external market, despite the high financing cost.
- 2. The insolvency of company here means the net assets are negative in a purely theoretical sense, that is, all equities of company (including intangible assets, human capital, etc.) are exhausted. In reality, insolvency of company does not necessarily lead to bankruptcy, because the company still has the value of existence, i.e. having net assets, for example, the assessed value of intangible assets of company, and so on.
- 3. Note: Coefficient 20 in $f(I) = 20\sqrt{I}$ has no essential meaning, and the purpose of using 20 is merely for the convenience of unitization – making the value of f(I) - I maximum when I = 100.

4. Parameters are selected based on the following: it can be found from the data on the interest-bearing debt/total capital input of listed companies in 2012, except financial companies and companies titled with ST, the ratio of about 95 percent's companies is lower than 90 percent, and that of 80 percent's companies is lower than 80 percent; therefore the paper selects 80 percent as the highest debt ratio limit of company, but this parameter will be adjusted and compared in the analysis below; considering that the income tax rate of company in force in China is 25 percent, τ = 25%; since 2009, the interest rate of short-term bank loans in China fluctuates between 4.86 percent and 6.10 percent, so r_D = 5% in the paper (besides, the interest rate of corporate bonds is maintained at about 5 percent in recent years); since 2009, the average annual rate of return in Shanghai Composite Index of China is approximately 10 percent, so r_E = 10%; because the paper adopts a three-stage model, it is not easy for parameter a to find a corresponding parameter in the real economy; therefore μ₂ = μ₃ = 1 and σ₂ = σ₃ = 0.3 for the time being here, and special parameter changes are conducted for these two variables below.

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Further reading

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